Study of Spacecraft Hover and Translation Modes Above the Lunar Surface

W. F. KELLER*
Northrop Corporation, Hawthorne, Calif.

Hover and translation in the absence of a finite atmosphere, such as with the lunar environment, preclude the use of aerodynamic lifting devices and require the use of rocket thrust. Three modes of thrust application and vehicle orientation through which this may be achieved are analyzed. Particular application is to lunar landing; however, equations are derived in a general form, applicable to any case where aerodynamic effects can be neglected. Performance of the modes is optimized and compared on the basis of propellant consumption and total system weight. The optimum mode is one in which the thrust vector required for translation is obtained by tipping the vehicle and aligning the engine with the vehicle centerline. Optimum values of vehicle tilt angle and translation time have been found to exist; these values are a function of translation requirements.

Nomenclature

a = acceleration, ft/sec²
F_h = horizontal thrust vector, lb
F_r = resultant thrust vector, lb
F_v = vertical thrust vector, lb
g_c = Earth gravity constant, ft/sec²
g = local acceleration of gravity, ft/sec²
I_{sp} = propellant specific impulse, lb-sec/lb
I_{tot} = total impulse, lb-sec
S_3 = total distance translated, ft
T = design thrust of engine, lb
t_{tot} = total hover and translation time, sec
t_1 = acceleration time, sec
t_2 = time-to-start of deceleration, sec
t_3 = total translation time, sec
V_0 = initial velocity, fps
WF = weight function, lb-sec/lb
W_{ip} = propulsion system inert parts weight, lb
W_0 = vehicle weight, lb
W_p = propellant weight, lb
W_r = propulsion system weight, lb
W_{eng} = engine weight, lb
θ = thrust vector angle, deg

Introduction

PREVIOUS studies[1, 2] have shown the existence of optimum conditions for minimum propellant consumption during vehicle translation parallel to the lunar surface. This study represents a more graphical approach in which the influence of over-all propulsion system weight is considered along with the effect of operating off of the optimum design points. In particular, consideration of operation when translation duration is preselected (or determined by maximum desired velocity), and not subject to optimization, is included. This appears appropriate, since a primary purpose of translation may be terrain observation at a particular velocity. Control considerations are beyond the scope of this study; an excellent closed-loop solution is provided in Ref. 3.

Analytical equations have been developed in general form, applicable to any planet (but neglecting aerodynamic effects) before application to the lunar case. The equations cope with an initial horizontal velocity and provide for a period of hover before and/or after translation. Altitude is assumed constant at 100 ft above the surface. The simplifying assumption is made that the propellant consumed during hover and translation is sufficiently small (less than 10%) to permit analysis on the basis of constant vehicle mass. In this case, the propellant consumed during hover only, in terms of vehicle weight fraction, is given by

$$W_p/W_0 = gt/g_c I_{sp} \qquad (1)$$

The preceding equation may be used to provide a feel for the maximum hover and translation durations, which should be studied. A reasonable quantity of propellant to devote to this operation appears to be 5% of the vehicle weight. On this basis, Table 1 shows the duration for pure hover on the moon, Mars, and Earth for the storable propellant, nitrogen tetroxide/50% unsymmetrical dimethyl hydrazine (UDMH)-50% hydrazine, and for the cryogenic propellant, oxygen/hydrogen. These durations will, of course, be reduced because of propellant consumption during translation. Total durations for hover and translation of 30, 60, and 90 sec have been selected for study. Translational range has been limited to a maximum of 3000 ft. Essentially three basic modes of rocket thrust chamber orientation and use are possible.† Descriptions are provided in Fig. 1 and in the following paragraphs.

Mode 1: Vehicle tipped to provide an engine thrust vector coinciding with the required resultant in direction and magnitude. Tipping can be accomplished through engine gimballing and/or use of reaction jets. A throttling engine is required.

Mode 2: Fixed engines, one providing a horizontal vector through the center of gravity, and one a vertical vector. Throttling or gimballing the engine is not a necessity, nor is canting the vehicle or engine required.

Mode 3: Fixed engines canted through the vehicle center of gravity. Thrust chambers differentially throttled to

Table 1 Hover duration

Location	Hover time at $W_p/W_0 = 0.05$, sec[a] N_2O_4/50-50	O_2/H_2
Moon	94	124
Mars	38	50
Earth	12	16

[a] Performance based on optimum nozzle expansion.

† Reference 1 shows a fourth mode, a ballistic transfer, to result in less propellant consumption; however, it does not lend itself well to visual observation.

Presented as Preprint 64-341 at the 1st AIAA Annual Meeting, Washington, D. C., June 29-July 2, 1964; revision received December 16, 1964. The author is pleased to acknowledge the encouragement and advice rendered by G. Chuck of Northrop Ventura and D. Houghtaling of Douglas Aircraft Company, Inc.

* Propulsion Specialist. Member AIAA.

translate: for maximum acceleration one is throttled to zero, the other to full thrust. Result during translation is similar to mode 1, but vehicle is not tipped. Engines are not gimballed, and stepped throttling may be used.

Kinematic aspects can be studied without regard for mode, and the results can be applied to all of the types. Analysis of the modes is on a weight basis: first, on the basis of propellant consumed, and second, on the basis of propulsion system weight for the particular application. Comparison of the modes is on a similar basis.

Kinematics

The relationships between distance, time, acceleration, and velocity are investigated in this section. Figure 2 portrays the velocity and acceleration-time relationships in general. The critical time parameters are the total allowable translation time t_3 and the acceleration time t_1. The decelerating capability of the vehicle is assumed to be equal to the accelerating capability; however, the time increments are not equal unless the initial velocity V_0 is zero. Using basic kinematic equations, the total horizontal distance traveled may be expressed as

$$S_3 = -at_1^2 + (at_3 - V_0)t_1 + V_0t_3 - V_0^2/2a \qquad (2)$$

where a is the magnitude of the horizontal acceleration (or deceleration). If V_0 is zero, this becomes

$$t_1^2 - t_1t_3 + S_3/a = 0 \qquad (3)$$

A slightly more complicated but workable expression for t_1 can be obtained if $V_0 \neq 0$. It will be noted from the previous sketch that the upper limit on t_1 occurs when it is equal to $t_3/2$, i.e., the vehicle accelerates for half of the time and decelerates for the remaining half. Minimum acceleration also occurs in this case, but the peak velocity is a maximum. With higher acceleration, a coast period $t_2 - t_1$ occurs before deceleration. The minimum t_1 value is limited by the accelerating force available. It is, of course, necessary to limit maximum acceleration to a value within an astronaut's physical capability and to limit maximum velocity to a value that permits visual observations and terrain avoidance.

If the angle θ is defined as the angle that the resultant thrust force on the vehicle makes with the vertical, then

$$\tan\theta = F_h/F_v \qquad (4)$$

The vertical component of this thrust vector F_v is equal to the local vehicle weight W_0g/g_c. Also

$$a = F_hg_c/W_0 \qquad (5)$$

or

$$a = g\tan\theta \qquad (6)$$

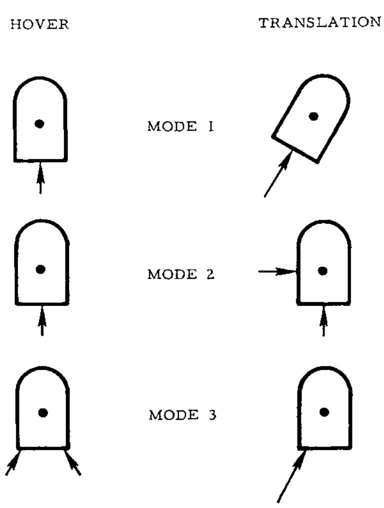

Fig. 1 Hover and translation modes.

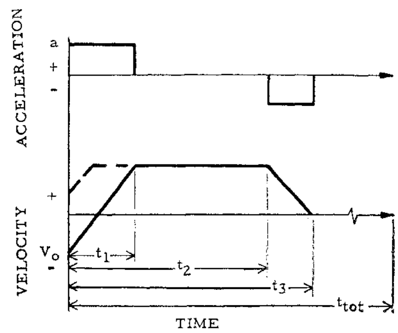

Fig. 2 Velocity and acceleration-time profiles.

Analysis of the Modes

Analysis and comparison of the modes are based on the study of propellant consumption and the resultant hover and translation propulsion system weight in the form of ratios to vehicle weight. System weight ratios are directly related to the propellant and system design selected and somewhat to the vehicle weight. Thus, although the propellant comparison holds as a general relationship, the results of the propulsion system comparison pertain only to the application and the following assumptions: vehicle weight, 11,000 lb; propulsion system, pressure fed at 150-psi chamber pressure; propellant, nitrogen tetroxide/50% UDMH-50% N_2H_4; propellant specific impulse, 310 lb-sec/lb (average throttled); inert propulsive to propellant weight ratio, 0.0816 (tanks, pressurization system, etc.); and thrust to engine weight ratio, 44.

Mode 1

As noted, the mode 1 engine (or engines) is throttled while the vehicle is hovering or coasting, and thrust is increased to the desired resultant value during translation.

Propellant consumption

Propellant weight that is consumed may be defined in terms of total impulse and specific impulse as

$$W_p = I_{tot}/I_{sp} \qquad (7)$$

Referring to Fig. 2, and noting that the resultant thrust vector F_r acts during acceleration and only the vertical vector F_v acts during periods of coast or hover, the equation for total impulse may be written as

$$I_{tot} = F_rt_1 + F_v(t_2 - t_1) + F_r(t_3 - t_2) + F_v(t_{tot} - t_3) \qquad (8)$$

The time-to-start of deceleration t_2 may be eliminated using the relation

$$t_2 = t_3 - t_1 - V_0/a \qquad (9)$$

Using the kinematic relations previously derived and substituting in Eqs. (7) and (8),

$$\frac{W_p}{W_0} = \frac{g}{g_cI_{sp}}\left[\left(2t_1 + \frac{V_0}{g\tan\theta}\right)\left(\frac{1}{\cos\theta} - 1\right) + t_{tot}\right] \qquad (10)$$

which is the general design equation for this mode. The case of $V_0 = 0$ and $t_{tot} = t_3$ has been selected for analysis. In this case, Eq (10) becomes

$$W_p/W_0 = (g/g_cI_{sp})[(2t_1/\cos\theta) - 2t_1 + t_3] \qquad (11)$$

Equation (3) permits solution of this equation in terms of thrust vector angle, translation duration, and distance translated. A typical result of this solution is shown in Fig. 3 as a plot of propellant weight function as influenced by thrust vector angle. The weight function WF is a convenient means of describing the propellant consumption without reference to a

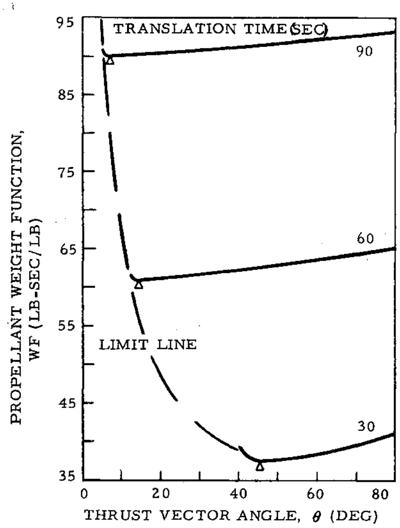

Fig. 3 Mode 1: influence of thrust vector angle on propellant weight function for a 1000-ft translation.

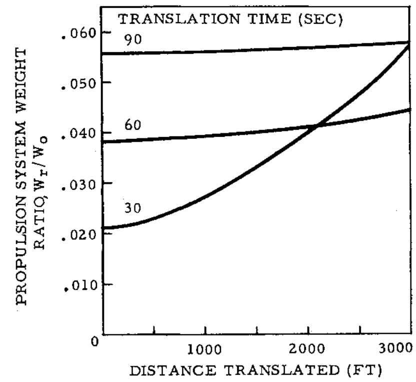

Fig. 5 Mode 1: effect of distance translated (at optimum angle) on ratio of propulsion system weight to vehicle weight.

particular vehicle or propellant and is equal to the terms within the brackets of Eq. (11). The limit line on the left of Fig. 3 denotes the points of minimum allowable acceleration. Figure 3 shows that minimum fuel consumption, for a required translation time, occurs at definite optimum values of thrust vector angle. These minimums may be obtained by differentiating Eq. (11) with respect to θ and setting the results equal to zero. The optimum values are plotted in Fig. 4 for lines of constant distance and time. Note the large thrust vector angles required to translate 2000 or 3000 ft in 30 sec. The curves intersect the ordinate axis at the value of weight function required to only hover for the durations noted.

System weight

An equation for the mode 1 propulsion system weight may be derived using the assumptions noted at the beginning of the section. Basically, the propulsion system weight is equal to the weight of propellant, inert parts (tankage, pressurization system, and plumbing), and engine, or

$$W_r = W_p + W_{ip} + W_{\text{eng}} \tag{12}$$

The inert parts are approximately proportional to the propellant weight; for the system assumed the ratio is estimated at 0.0816. Equation (12) may thus be written as

$$W_r = 1.0816\, W_p + W_{\text{eng}} \tag{13}$$

By dividing by the vehicle weight W_0 and substituting the weight function WF,

$$W_r/W_0 = 1.0816\, g(WF)/g_cI_{\text{sp}} + W_{\text{eng}}/W_0 \tag{14}$$

The engine-vehicle weight ratio is derived as

$$W_{\text{eng}}/W_0 = (T/W_0)/(T/W_{\text{eng}}) = g/g_c \cos\theta (T/W_{\text{eng}}) \tag{15}$$

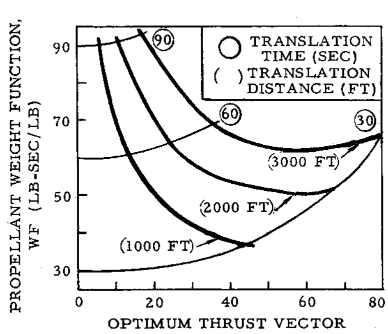

Fig. 4 Mode 1: propellant weight function vs optimum thrust vector angle.

Since the value of thrust-to-engine weight can be estimated, the engine-vehicle weight ratio may be determined for any value of θ. Equation (14) thus may be solved for the lunar mission, using the assumed value of I_{sp} and proper gravity constants. Because engine design (maximum) thrust level is a function of θ, each value of θ will represent a specific propulsion system design. Optimum values of thrust vector angle remain essentially the same as for the propellant weight function, except for translation distances of 2000 and 3000 ft in 30 sec. The rather large angles indicated in Fig. 4 for the weight function are lowered by the influence of engine weight ratio; Eq. (15) shows this fraction to be significant for large angles. Figure 5, a plot of system weight ratio vs distance translated at optimum angle, also reflects the influence of engine weight. System weights increase very rapidly with distance for a 30-sec translation time because of the influence of higher accelerations on both propellant consumption and engine weight.

Mode 2

As noted, mode 2 requires a separate engine thrusting horizontally through the vehicle center of gravity.

Propellant consumption

An equation may be written for total impulse in a manner similar to mode 1 as

$$I_{\text{tot}} = F_h t_1 + F_h(t_3 - t_2) + F_v\, t_{\text{tot}} \tag{16}$$

and, using the relations as in deriving Eq. (11),

$$W_p/W_0 = (g/g_cI_{\text{sp}})[(2t_1 + V_0/a)\tan\theta + t_{\text{tot}}] \tag{17}$$

With $V_0 = 0$ and no hover before or after translation, the equation becomes

$$W_p/W_0 = (g/g_cI_{\text{sp}})(2t_1 \tan\theta + t_3) \tag{18}$$

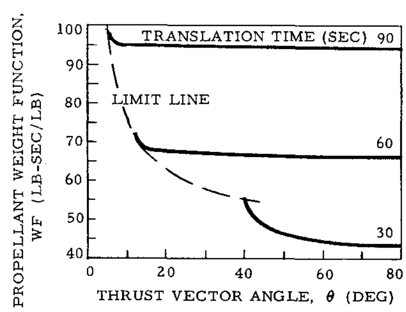

Fig. 6 Mode 2: influence of thrust vector angle on propellant weight function for a 1000-ft translation.

Propellant consumption effects may be studied as in mode 1. Typical results are presented in Fig. 6, which plots weight function vs thrust vector angle. It may be seen that an optimum point does not exist, and that propellant consumption is improved with higher acceleration. The limit line on the left denotes the point of minimum allowable acceleration.

System weight

Equation (14), derived for the previous mode, also applies here; however, the equation for engine weight differs in that

$$W_{\rm eng}/W_0 = (\tan\theta + 1)(g/g_c)(T/W_{\rm eng})^{-1} \qquad (19)$$

Solution for system weights is similar to mode 1. Optimum angles or accelerations now exist. The increase in engine weight with acceleration quickly over-balances the propellant weight saving and causes an optimum acceleration value to exist for each duration and distance. System weight ratios at these optimum values are plotted vs distance translated in Fig. 7. It will be noted that the weight ratio at 30 sec of translation climbs steeply with distance. As with mode 1, this is caused by the need for very large accelerations to achieve the distance within the allowed time, thus requiring thrusts (and engine weights) greatly in excess of longer duration designs.

Mode 3

Mode 3 requires two engines, both of which are throttled during hover and coast; one must be turned off, whereas the other operates at full thrust during lateral acceleration.

Propellant consumption

An equation that may be written for total impulse as in the previous modes is

$$I_{\rm tot} = F_r t_1 + 2F_r'(t_2 - t_1) + F_r(t_3 - t_2) + 2F_r'(t_{\rm tot} - t_3) \qquad (20)$$

where F_r' represents the full thrust of one engine during acceleration, and F_r represents the throttled thrust of each engine during coast. Solving for propellant weight ratio as before,

$$W_p/W_0 = (g/g_c I_{sp})(t_{\rm tot}/\cos\theta) \qquad (21)$$

thus obtaining an equation similar to those of the other two modes (10 and 17). With no hover before or after translation, the term t_3 is substituted for $t_{\rm tot}$. It may be observed that propellant is consumed at the same rate whether the vehicle translates or not. In this case, θ also represents engine cant angle. The influence of cant angle (or thrust vector angle) on the weight function is shown in Fig. 8. It will be noted that minimum propellant consumption occurs at the minimum cant angle capable of providing the necessary acceleration. Curves of weight function vs distance can be plotted, using these minimum values, with results similar to the other modes.

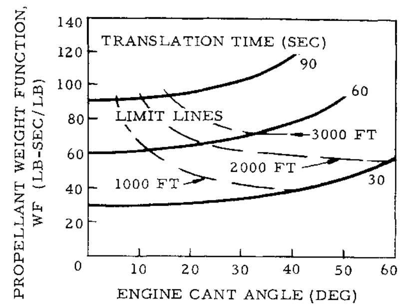

Fig. 8 Mode 3: influence of engine cant angle on propellant weight function.

System weight

System weights for this mode are determined similarly to mode 1 except that engine weights are twice as large for a given angle. This is because each of the two engines must be capable of providing the same thrust. Curves are again similar in form to the other modes.

Comparison of the Modes

Comparison of the modes is on the basis of propellant consumption and system weight.

Propellant Consumption

The propellant consumption, with respect to thrust vector angle, is illustrated for each of the modes in Fig. 9 for a translation distance and time of 1000 ft and 30 sec. It may be seen that modes 1 and 3 have a common value at the minimum acceleration point. At this point, the translation time is divided equally between accelerating and decelerating, and the thrust vectors of the two modes are identical. For accelerations greater than minimum, a coast period exists. Under these conditions, mode 3 becomes extremely inefficient, since both canted engines must operate and their horizontal thrust vectors cancel each other. Propellant consumption rises quite rapidly with acceleration, as larger cant angles are required and more horizontal thrust is lost during coast.

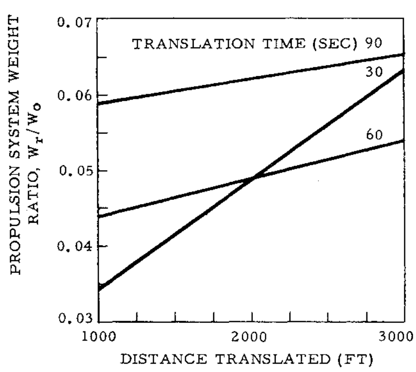

Fig. 7 Mode 2: effect of distance translated (at optimum acceleration) on ratio of propulsion system weight to vehicle weight.

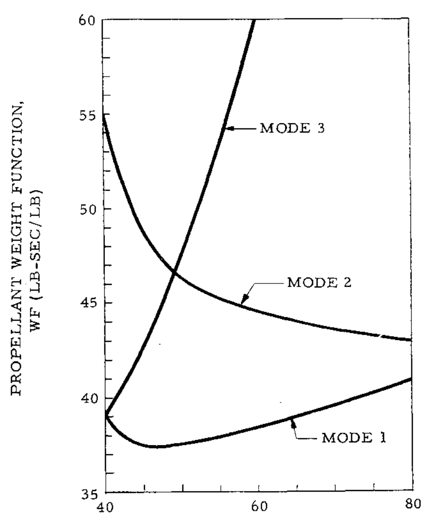

Fig. 9 System comparison: propellant weight function vs thrust vector angle (to translate 1000 ft in 30 sec).

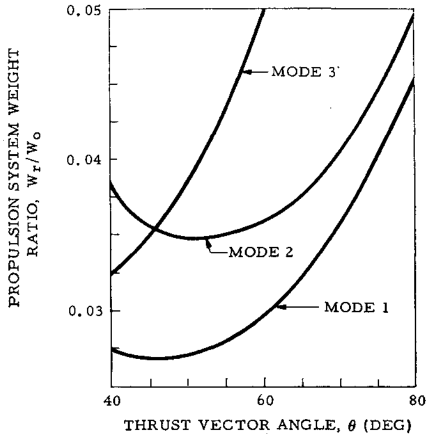

Fig. 10 System comparison: influence of thrust vector angle (to translate 1000 ft in 30 sec) on ratio of propulsion system weight to vehicle weight.

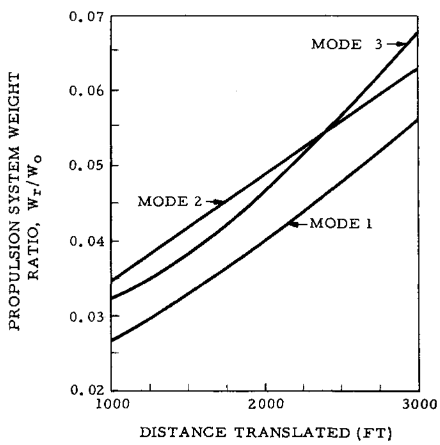

Fig. 11 System comparison: system weight ratio vs distance translated in 30 sec (at optimum angle).

Thrust vector relationships are the key to much of the behavior of the modes. For example, the explanation of the differing propellant consumption of modes 1 and 2 lies in the thrust vector application during acceleration. The horizontal and vertical thrusts of mode 1 combine vectorially to a value, upon which propellant consumption is based, that is less than the actual sum of the two. Since the horizontal and vertical vectors of mode 2 represent separate engines, they add directly and thus result in greater propellant consumption for the same resultant force.

System Weight

In Fig. 10, the propulsion system weight ratio is plotted against thrust vector angle for a typical translational requirement. It may be seen that mode 1 provides almost a 20% weight saving over the next lightest system (mode 3). As previously discussed, the curves terminate on the left, at the angle that provides the minimum acceleration possible for the given translational requirement. Figure 11 plots propulsion system-vehicle weight ratio (at optimum thrust vector angle) vs distance translated for a duration of 30 sec; similar plots occur for other durations. Mode 1 is superior to the other modes in all of the cases.

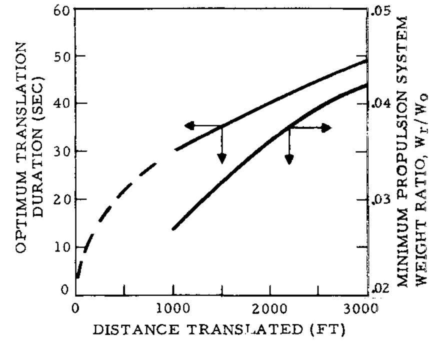

Fig. 12 Optimum translation durations and minimum system weight ratios.

Summary

Mode 1 (vehicle tipped to translate) provides both minimum fuel consumption and system weight and is the recommended method. Modes 2 and 3 have been considered because they offer the ability to translate with the vehicle vertical and because they minimize or eliminate the need for engine throttling and the accompanying reliability problem. However, study of mode 1 indicates that the vehicle tip angle need not be severe; also, the additional engines required by modes 2 and 3 are in themselves a detriment to reliability.

An additional advantage of mode 1 is that the lunar vehicle engine used for retrograde may be utilized for hover and translation as well. Since the retrograde engine must be capable of providing acceleration in excess of the lunar gravity, design thrust level probably does not need to be increased for the translation function. Thus the engine weight penalty included in this analysis is conservative.

Review of Figs. 4 and 5 indicates the existence of optimum translation times to travel a given distance, as well as optimum thrust vector angles. This is verified by Ref. 1, which also found that the angle corresponding to these minimum times, for minimum propellant consumption, is always 60° regardless of the distance translated. When the total system weight is considered or, in particular, the thrust chamber weight, this value is shifted to 45°. Optimum durations are also greater. Figure 12 shows the optimum durations and minimum system weight ratios for the distance studied. It may be seen that distances of up to 3000 ft may be translated at a propulsion system cost of less than 5% of vehicle weight.

References

[1] Sivo, J. N., Campbell, C. E., and Hamza, V., "Analysis of close lunar translation techniques," NASA-TR-R-126 (1962).

[2] Cheng, R. K. and Conrad, D. A., "Optimum translation and the brachistochrone," AIAA J. 1, 2845–2847 (1963).

[3] Conrad, D. A., "Minimum fuel closed loop translation," Joint IMS-SIAM-ONR-AIAA Symposium on Control and System Optimization, Monterey, Calif. (January 1964).